A DIFFUSION FLOW MODEL FOR CANAL FLOW NETWORKS

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none

INTRODUCTION

Understanding the behavior of large scale hydrologic systems such as the system in South Florida involves understanding the flow in conveyance systems consisting of large numbers of canals and flow control structures. Inflow into these systems can come from overland flow over wetland, urban or agricultural areas, ground water flow from neigboring areas, and tidal flows from points connected to the ocean. Coveyance systems constructed in the Everglades are closely connected with weltands and ground water systems. As a result, hydrologic systems such as the South Florida systems cannot be accurately simulated without properly representing a large number of canal elements of the conveyance system.

Flow in an open channel systems can be simulated by solving complete St. Venant equations. Diffusion flow equations which are derived by neglecting the inertia terms of the complete equations are also useful in the case of regional hydrologic models in which such terms are negligible. Ponce, et al. (1978) established conditions under which diffusion flow models can be solved instead of the complete equations. Diffusion flow models only solve continuity equation and the momentum equation teduces to a simple form. The total number of equations to be solved become halved when diffusion flow models are used. The ordinary differential equations resulting from the spatial discretization of the diffusions flow equations are discretized using a weighted

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implicit method. As in the case of UNET, BRANCH and many other models solving the complete equations, weighted implicit method methods provide control over stability and accuracy, and methods to handle stiffness. Stiffness is a problem with large networks having large and small canal discretizations together.

The sparse system of equations resulting from the discretization can be solved using freely and comercially available solvers. A large number of high performance linear equation solvers have recently become available because of the demand from many research and industrial applications using numerical analysis. These methods are capable of solving the equations using a munimum number of computions, depending on the extent of disturbance or stress imposed on the system during the time step. Iterative methods based on the preconditioned conjugate gradient methods have become the most popular linear solvers.

In this paper, a network model based on solving the diffusion equations is applied over part of the South Florida hydrologic system. to verify the accuracy of the model.

GOVERNING EQUATIONS

Gradually varied unsteady overland flow is explained using the depth averaged flow equations commonly referred to as Saint Venant equations. The continuity equation is

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} + q_{ea} = 0 \tag{1}$$

in which, x is the distance measured along the canal; A = flow cross sectional area; Q = discharge through A; q_{ea} = overland flow entering into the canal per unit length. Rainfall and evapo-transpiration over the canal is assumed to be taking place in the overland flow area. The momentum equations is

$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x}\left(\beta \frac{Q^2}{A}\right) + gA(\frac{\partial H}{\partial x} + S_f) = 0 \tag{2}$$

in which, S_f = friction slope in x direction; H = water level; β = momentum correction coefficient. After neglecting the first three terms contributing to inertia effects, the momentum equation in reduces to $\frac{\partial H}{\partial x} = -S_f$ in which, $H = h + z$ = water level above a datum; z = bottom elevation above datum. A general form of the Manning's equation is written as $V = \frac{1}{n} R^{\gamma} S_f^{\lambda}$ in which $R = A/P$ = hydraulic radius; P = wetted canal perimeter; n = Manning's coefficient when $\gamma = 2/3$ and $\lambda = 1/2$; S_f = friction slope. Akan and Yen (1981), Hromadka et al. (1987), and others showed that flow velocity components u can be expressed in the following form using Manning's equations.

$$Q = -K \frac{\partial H}{\partial x} \tag{3}$$

in which, K can be expressed for the Manning's equation as

$$K = \frac{1}{n} A R^{\gamma} S_n^{\lambda - 1} \quad \text{for} \quad \lambda \geq 1, \quad |S_n| > \delta \tag{4}$$

$$K = \frac{1}{n} A R^{\gamma} S_n^{1 - \lambda} \quad \text{for} \quad \lambda < 1, \quad |S_n| > \delta \tag{5}$$

$$K = 0 \quad \text{for} \quad \lambda < 1 \quad \text{and} \quad |S_n| \leq \delta \quad \text{or} \quad R = 0 \tag{6}$$

The condition $R = 0$ facillitates wetting and drying. δ is used to bound K within finite limits. $\delta \approx 1.0 \times 10^{-10}$ is used in the study for single precision runs. $\lambda \approx 1$ gives conditions in the wetlands closer to laminar flow. K is useful in linearizing and simplifying the diffusion flow equation. The continuity equation Eq. 1 can be expressed using Eq. 3 as

$$\frac{\partial A}{\partial t} = \frac{\partial}{\partial x} K \frac{\partial H}{\partial x} + q_{ae} \tag{7}$$

Methods available to solve the diffusion equations can be used to solve this non-linear partial differential equation.

NUMERICAL METHOD

The canal system is discretized into small canal sections, each having uniform cross sectional properties. Each canal section is connected to the next section at a node. In order to obtain a finite volume type formulation, the governing equation is expressed in the following integral form over a control volume surrounding a canal element.

$$\frac{\partial}{\partial t}\int_{cv} A dv - \int_{cv}\frac{\partial Q}{\partial x}dx + \int_{cv} q_{ae} dv = 0 \tag{8}$$

For a canal element of plan area $B\Delta x$, the continuity equation can be expressed as

$$\frac{\partial H}{\partial t} = \frac{1}{B\Delta x}[\sum Q_{in} - \sum Q_{out} + q_{ae}] = 0 \tag{9}$$

in which, Q_{in} and Q_{out} are flows in and out from neighboring canal sections.

When two canal sections meet at the node, the flow from one canal section to the other is computed using the friction contributions of the two canal sections. If the water levels of two canal sections i and j are H_i and H_j, the discharge between the sections can be expressed as

$$Q = K_{i,j}\Delta H \tag{10}$$

in which $\Delta H = H_i - H_j$ and

$$\frac{1}{K_{i,j}} = \sqrt{0.5(\frac{\Delta x_i}{K_i^2} + \frac{\Delta x_j}{K_j^2})\Delta H} \tag{11}$$

K_i, K_j are computed using Eq. 4, 5 and 6 over sections i and j. The equation of mass balance for section i can be expressed as

$$\frac{\partial H_i}{\partial t} = \frac{1}{B_i\Delta x_i}[\sum_{j=1}^{nd} K_{i,j}H_j + q_{ae,i}] = 0 \tag{12}$$

in which, $j = 1, 2 \ldots nd$ are the sections connected to section i.

If more than one canal section meet at a node, the resistance to flow from any canal section to the other is computed by linearizing the resistance equations. If n river sections meet at the node, the discharge through any section i in to the joint is

$$Q_i = K_i\frac{H_i - C}{0.5\Delta x_i} \tag{13}$$

Since the sumation of all the discharges through the sections into a joint is zero, the following expression can be obtainrd for the water level at the joint.

$$C = \frac{\sum_{i=1}^{n} \frac{K_i H_i}{\Delta x_i}}{\sum_{i=1}^{n} \frac{K_i}{\Delta x_i}} \tag{14}$$

With the knowledge of C, it is possible to compute the discharge through any of the pipe sections in the network.

$$Q_i = \frac{2K_i K_j}{D \Delta x_i \Delta x_i} \quad \text{for} \quad j = 1, n, \ j \neq i \tag{15}$$

in which,

$$D = \sum_{i=1}^{n} \frac{K_i}{\Delta x_i} \tag{16}$$

Equation 15 can be used to compute the following expression for discharge between canal sections i and j

$$K_{ij} = \frac{2K_i K_j}{D \Delta x_i \Delta x_j} \tag{17}$$